Separable First-Order Differential Equations Differential Equations X. Du

- **Separable Differential Equation:** $\frac{dy}{dx} = g(x)h(y)$
  - Example: $\frac{dy}{dx} = xy + 2x$
  - 1) Separate variables. 2) Integrate. 3) Solve for *y*.
  - Don't forget equilibrium solution!
- **First-Order Autonomous Ordinary Differential Equations**: $\frac{dy}{dx} = f(y)$
  - Slope does not depend on *x*.
  - All horizontal lines are isoclines.
  - Integral curves can be shifted horizontally and they will still be an integral curve
  - Solve as a separable first-order differential equation.
  - **Critical point**: *y* values such that $f(y) = 0$.
  - **Equilibrium solutions:**
    - Where slope is zero.
    - Occur at critical points. If $f(y_o) = 0$, then $y = y_o$ is an equilibrium solution
    - Serve as "boundaries" of integral curves (assuming $f(y)$ is differentiable).
    - Determine end behavior.
    - **Stable equilibrium solution**: Solutions tend to converge to this solution
    - **Unstable equilibrium solution**: Solutions diverge from this solution
    - **Semi-stable equilibrium solution**: On one side, solutions tend to converge to this solution. On the other side, solutions tend to diverge.
  - **Exponential Growth and Decay**
    - $\frac{dy}{dx} = ky$ Solution: $y = Ce^{kx}$
    - Applications:
      - Newton's Law of Cooling
      - Radioactive Decay
      - Continuously compounded interest
      - Discharging a capacitor
  - **Logistical Growth and Decay**
    - $\frac{dy}{dx} = ky\left(1 - \frac{y}{L}\right)$ Solution (use partial fractions): $y = \frac{L}{1 + Ce^{-kx}}$
    - Applications: Population growth with carrying capacity
- **Homogeneous Differential Equations:**
  - If you substitute *tx* and *ty* in for *x* and *y* in the differential equation, then $f(tx, ty) = t^n f(x, y)$
  - Example: $\frac{dy}{dx} = \frac{3x + 2y}{5x}$
  - $y = vx$ $dy = vdx + xdv$ (product rule) Use these substitutions.
  - Substitute into original differential equation. Solve as separable.